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169. Proposed by JOHN M. COLAW, A. M., Monterey, Va.

$$\text{Solve } x^2 + y + z = a \dots (1).$$

$$x + y^2 + z = b \dots (2),$$

$$x + y + z^2 = c \dots (3).$$

I. Solution by L. C. WALKER, A. M., Graduate Student, Leland Stanford Jr. University, Cal.

Eliminating z between (1) and (2); x between (2) and (3); y between (3) and (1), we obtain

$$(x-y)(x+y-1) = a-b \dots (4),$$

$$(y-z)(y+z-1) = b-c \dots (5), \text{ and}$$

$$(z-x)(z+x-1) = c-a \dots (6).$$

By addition of corresponding members of (4), (5), and (6), we have

$$(x-y)(x+y-1) + (y-z)(y+z-1) + (z-x)(z+x-1) = 0 \dots (7).$$

It is easy to see that either $x=y=z \dots (8)$, or $x+y-1=y+z-1=z+x-1=0 \dots (9)$ will satisfy (7).

Now from (8) and (1); from (8), (2); from (8), (3); from $y+z=1$, (1); from $z+x=1$, (2); and from $x+y=1$, (3); we obtain $x=y=z=-1 \pm \sqrt{a+1}$, $-1 \pm \sqrt{b+1}$, or $-1 \pm \sqrt{c+1}$, and

$$x = \pm \sqrt{a-1},$$

$$y = \pm \sqrt{b-1},$$

$$z = \pm \sqrt{c-1}.$$

II. Solution by G. B. M. ZERE, A. M., Ph.D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let $x+y+z=s$. Then

$$x^2 + s - x = a, \text{ or } x = \frac{1}{2} \pm \sqrt{a-s+\frac{1}{4}} \dots (1),$$

$$y^2 + s - y = b, \text{ or } y = \frac{1}{2} \pm \sqrt{b-s+\frac{1}{4}} \dots (2),$$

$$z^2 + s - z = c, \text{ or } z = \frac{1}{2} \pm \sqrt{c-s+\frac{1}{4}} \dots (3).$$

$$(1)+(2)+(3) \text{ gives } s = \frac{3}{2} \pm \{\sqrt{a-s+\frac{1}{4}} + \sqrt{b-s+\frac{1}{4}} + \sqrt{c-s+\frac{1}{4}}\}.$$

$$\text{Let } s - \frac{1}{2} = m \dots (4).$$

$$\text{Then } m - \frac{1}{4} \mp \sqrt{a-m} = \pm \{\sqrt{b-m} + \sqrt{c-m}\} \dots (5).$$

Squaring (5),

$$m^2 - \frac{3}{2}m + \frac{3}{16} + a - b - c = 2\sqrt{\{[b-m][c-m]\}} \pm 2(m - \frac{1}{4})\sqrt{a-m} \dots (6).$$

Squaring (6),

$$m^4 + m^3 - 2(a+b+c + \frac{1}{8})m^2 + 7(a+b+c + \frac{1}{16})m + a^2 + b^2 + c^2 + \frac{3}{8}m$$

$$- 2(ab+ac+bc) - \frac{2}{8}(a+b+c) = \pm 8(m - \frac{1}{4})\sqrt{\{[a-m][b-m][c-m]\}}$$

$$\text{or } m^4 + m^3 - Am^2 + Bm + C = \pm 8(m - \frac{1}{4})\sqrt{\{[a-m][b-m][c-m]\}}, \text{ suppose.}$$

Squaring this last equation

$$\begin{aligned}
 m^8 + 2m^7 + (1-2A)m^6 + (64+2B-2A)m^5 + [160-64(a+b+c) + A^2 \\
 + 2C+2B]m^4 + [64(ab+ac+bc) - 160(a+b+c) + 100+2C-2B]m^3 \\
 + [160(ab+ac+bc) - 100(a+b+c) - 64abc + B^2 - 2AC]m^2 \\
 + [100(ab+ac+bc) + 160abc + 2BC]m + C^2 - 100abc = 0.
 \end{aligned}$$

This equation gives m . m in (4) gives s , and s in (1), (2), (3) gives x, y, z .

Also solved by **MARCUS BAKER**, who combines 169 and 173 under one solution.

170. Proposed by **S. F. NORRIS**, Baltimore City College, Baltimore, Md.

Find by strictly quadratic methods at least one set of values of x and y in the equations $x^2y^2+x=a=38$, and $xy+y^2=b=15$.

Comments and Analysis by **MARCUS BAKER**, Washington, D. C.

The values of xy and of x from the second equation substituted in the first one give an equation in y ; and similarly the values of xy and of y^2 from the second equation substituted in the first one give an equation in x . The two equations resulting from this elimination are

$$\begin{array}{rcl}
 x^5 + (b^2-a)x^4 + 2bx^3 + (1-2ab)x^2 - 2ax + a^2 = 0 \dots (1), \\
 y^5 \qquad \qquad - 2by^3 - \qquad \qquad y^2 + (b^2-a)y + b = 0 \dots (2),
 \end{array}$$

equations of the fifth degree and of course insoluble by quadratics. Restoring numbers, the equations become

$$\begin{array}{rcl}
 x^5 + 187x^4 + 30x^3 - 1139x^2 - 76x + 1444 = 0 \dots (3). \\
 y^5 \qquad \qquad - 30y^3 - \qquad \qquad y^2 + 187y + 15 = 0 \dots (4).
 \end{array}$$

It is obvious by inspection of the original equations that $x=2$ and $y=3$ are a pair of roots; therefore (3) is exactly divisible by $x-2$ and (4) is similarly divisible by $y-3$. Performing this division there results

$$\begin{array}{rcl}
 x^4 + 189x^3 + 408x^2 - 323x - 722 = 0 \dots (5), \\
 y^3 + 3y^2 - 21y - 64 = 0 \dots (6),
 \end{array}$$

equations of the fourth degree. Can these equations be solved by *strictly quadratic methods*? The answer depends on the meaning of the italicized words. Every quartic is solvable after the manner of quadratics in terms of an auxiliary involved in a cubic equation; and every cubic is solvable after the manner of quadratics in terms of an auxiliary involved in a quadratic equation; and every quadratic is solvable in terms of an auxiliary involved in a simple equation. In a certain sense, therefore, *every* biquadratic is solvable by quadratics. If by *strictly quadratic methods* it is implied that the auxiliary involved in the cubic is